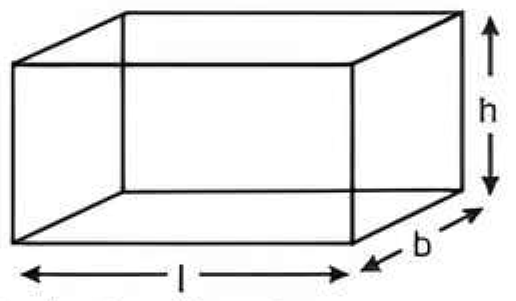
12 Surface Areas and Volumes

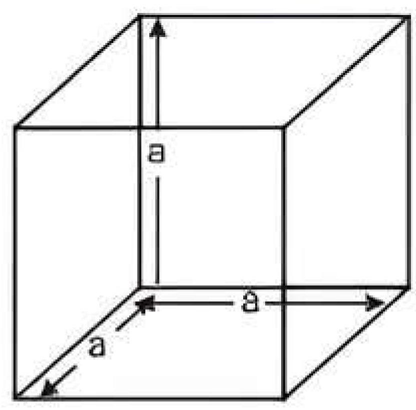
Fastrack Revision

- **Cuboid:** A cuboid has six faces. Let the length of a cuboid be 'l', breadth be 'b' and height be 'h'.



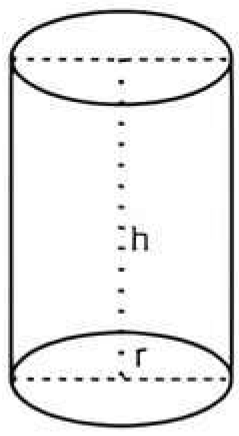
1. Volume $= l \times b \times h$ cubic units
2. Lateral Surface Area (LSA)
   = Area of the four walls of a room
   $= 2(l+b) \times h$ square units
3. Total Surface Area (TSA) $= 2(lb + bh + hl)$ square units
4. Length of the diagonal $= \sqrt{l^2+b^2+h^2}$ units

- **Cube:** A cube has six identical faces. Let each edge of a cube be 'a'.



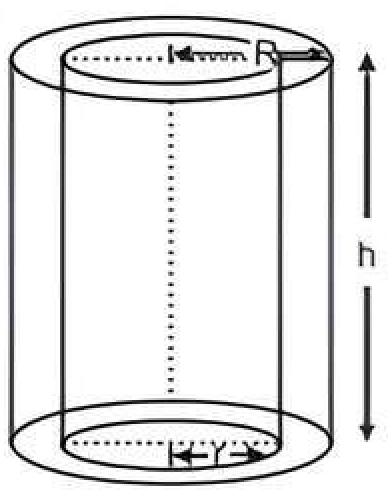
1. Volume = $(\text{Side})^3 = a^3$ cubic units
2. Lateral Surface Area (LSA) $= 4a^2$ square units
3. Total Surface Area (TSA) $= 6a^2$ square units
4. Diagonal $= a\sqrt{3}$ units

- **Right Circular Cylinder:** A cylinder has one curved face and two plane circular faces. Let 'r' be the base radius and 'h' be the vertical height of a cylinder.



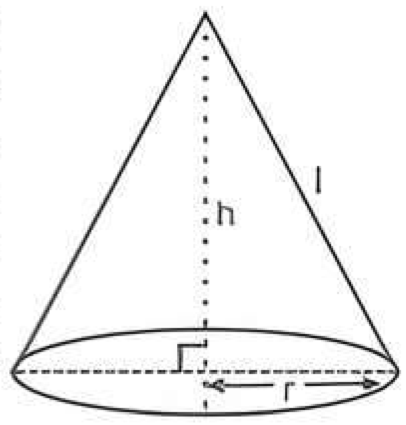
1. Volume = Area of base × Height
   $= \pi r^2 h$ cubic units
2. Curved Surface Area (CSA)
   = Circumference of base × Height
   $= 2\pi rh$ square units
3. Total Surface Area (TSA) = CSA + Area of two ends
   $= 2\pi rh + 2\pi r^2 = 2\pi r(h+r)$ square units

- **Hollow Cylinder:** Solids like pipes are in the shape of hollow cylinders. Let 'R' be the external radius, 'r' be the internal radius and 'h' be the height of a hollow cylinder.



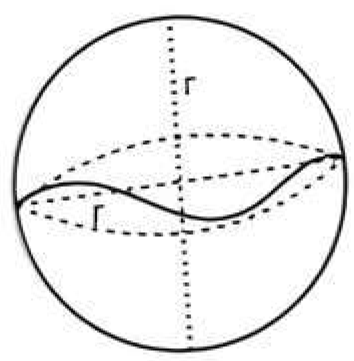
1. Area of cross-section
   $= \pi(R^2 - r^2)$ square units
2. Volume of material
   $= \pi R^2 h - \pi r^2 h$
   $= \pi h(R^2 - r^2)$
   $= \pi h(R+r)(R-r)$ cubic units
3. Curved Surface Area (CSA) = External surface area + Internal surface area
   $= 2\pi Rh + 2\pi rh = 2\pi h(R+r)$ square units
4. Total Surface Area (TSA)
   = Curved surface area of hollow cylinder + Area of top and base rings
   $= 2\pi h(R+r) + 2\pi(R^2 - r^2)$
   $= 2\pi(R+r)(R-r+h)$ square units
5. Thickness of cylinder $= R - r$ units

- **Right Circular Cone:** A cone has one curved face and one plane circular face. Solids like joker's cap, funnel, softy ice-cream, etc. are in the shape of cone. Let 'r' be the radius, 'h' be the vertical height and 'l' be the slant height of a right circular cone.



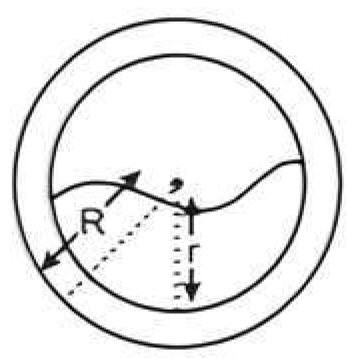
1. Volume $= \frac{1}{3}\pi r^2 h$ cubic units
2. Curved Surface Area (CSA) $= \pi rl$ square units
3. Total Surface Area (TSA)
   = CSA + Area of the base
   $= \pi rl + \pi r^2$
   $= \pi r(l + r)$ square units
4. Slant height $(l) = \sqrt{h^2+r^2}$ units

- **Sphere:** A sphere has only one curved surface. Let 'r' be the radius of sphere.



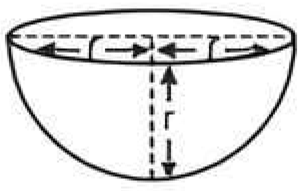
1. Volume $= \frac{4}{3}\pi r^3$ cubic units
2. Curved Surface Area (CSA)
   = Total Surface Area (TSA) $= 4\pi r^2$ square units

- **Spherical Shell (Hollow Sphere):** Let 'R' be the external radius and 'r' be the internal radius.



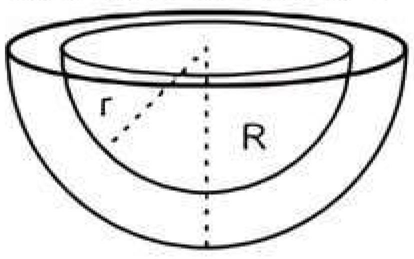
1. Volume of material
   = Outer volume − Inner volume
   $= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$
   $= \frac{4}{3}\pi(R^3 - r^3)$ cubic units
2. Thickness $= (R - r)$ units
3. Outer surface area $= 4\pi R^2$
4. Inner surface area $= 4\pi r^2$

- **Hemisphere:** A plane through the centre of a sphere cuts it into two equal parts. Each part is called a hemisphere. Let 'r' be the radius of hemisphere.



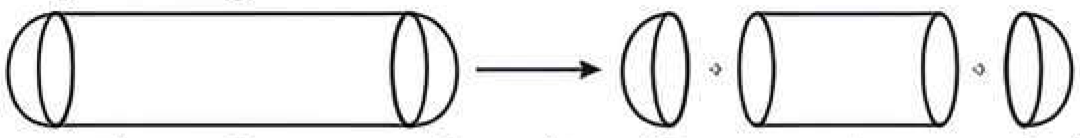
1. Volume $= \frac{2}{3}\pi r^3$ cubic units
2. Curved Surface Area (CSA)
   $= 2\pi r^2$ square units
3. Total Surface Area (TSA)
   = CSA + Area of the base
   $= 2\pi r^2 + \pi r^2 = 3\pi r^2$ square units

▶ **Hemispherical Shell:** Let 'R' be the external radius and 'r' be the internal radius.



1. Area of base $= \pi(R^2 - r^2)$ square units
2. Volume of material
   $= \frac{2}{3}\pi(R^3 - r^3)$ square units

▶ **Surface Area and Volume of Combination of Solid Figures:** Supposed combined figure of hemisphere and cylinder is given below:



1. The surface area of combined figure is the sum of all areas that are visible, (*i.e.*, generally the sum of all the curved surface of the individual figure.)
   Surface area of combined figure
   = CSA of hemisphere + CSA of cylinder
   + CSA of hemisphere.
2. The volume of combined figure is the sum of all the volumes of the individual figure.
   Volume of combined figure
   = Volume of hemisphere + Volume of cylinder
   + Volume of hemisphere.

Knowledge BOOSTER

1. *The maximum length of rod that can be fitted in a cuboid (or cube) is equal to the length of diagonal.*
2. *When we adjoin the two or more cubes, only the length of cuboid changes, but breadth and height remain constant.*
3. *Cost = Area × Rate*
4. *Density* $= \frac{Mass}{Volume}$
5. **Important Conversions of Units**
   - $1\ m^3 = 1000\ L$
   - $1\ m^3 = 1\ kL$
   - $1\ L = 1000\ cm^3$
   - $1\ mL = 1\ cm^3$
   - $1\ mm = \frac{1}{10}\ cm$
   - $1\ km/hr = \frac{5}{18}\ m/s$
   - $1\ m/s = \frac{18}{5}\ km/hr$
   - $1\ hectare = 10,000\ m^2$

Practice Exercise

Multiple Choice Questions

Q 1. The shape of a *gilli*, in the *gilli-danda* game is a combination of: [NCERT EXEMPLAR]
a. two cylinders
b. one cone and one cylinder
c. two cones and one cylinder
d. two cylinders and one cone

Q 2. A Surahi is the combination of: [NCERT EXEMPLAR]
a. a sphere and a cylinder
b. a hemisphere and a cylinder
c. two hemispheres
d. a cylinder and a cone

Q 3. A maximum diameter of sphere is carved out from the cube of edge 6 cm. The diameter of sphere is:
a. 3 cm b. 6 cm c. 12 cm d. 9 cm

Q 4. A cylinder and a cone have same base and same height. The ratio of their volumes is:
a. 3 : 1 b. 1 : 3 c. 2 : 3 d. 3 : 2

Q 5. The length of the longest pole that can be kept in a room (12m × 9 m × 8 m) is:
a. 17 m b. 19 m c. 21 m d. 29 m

Q 6. The sum of the length, breadth and height of a cuboid is $6\sqrt{3}$ cm and the length of its diagonal is $2\sqrt{3}$ cm. The total surface area of the cuboid is:
a. $48\ cm^2$ b. $72\ cm^2$
c. $96\ cm^2$ d. $108\ cm^2$

Q 7. If two cubes of edge 3 cm each are joined end to end, then the surface area of the resulting cuboid is:
a. $90\ cm^2$ b. $95\ cm^2$ c. $92\ cm^2$ d. $94\ cm^2$

Q 8. What is the total surface area of a solid hemisphere of diameter 'd'? [CBSE 2023]
a. $3\pi d^2$ b. $2\pi d^2$ c. $\frac{1}{2}\pi d^2$ d. $\frac{3}{4}\pi d^2$

Q 9. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is: [NCERT EXEMPLAR]
a. $3\pi r^2$ b. $6\pi r^2$ c. $3\pi r^2$ d. $8\pi r^2$

Q 10. Suppose h and r be the height and radius of cylinder. If cylinder is cut from the middle part, then the volume of the remaining part of cylinder is:
a. $\pi r^2 h$ b. $\frac{\pi r^2 h}{2}$
c. $2\pi r^2 h$ d. None of these

Q 11. The volume of a right circular cone whose area of the base is $156\ cm^2$ and the vertical height is 8 cm, is: [CBSE 2023]
a. $2496\ cm^3$ b. $1248\ cm^3$
c. $1664\ cm^3$ d. $416\ cm^3$

Q 12. Suppose height and radius of a solid cylinder are 15 cm and 6 cm. A cone is carved out from a cylinder the maximum height of cone is:
a. 6 cm b. 15 cm c. 13 cm d. 17 cm

Q 13. The volumes of two spheres are in the ratio 64 : 27. The ratio of their surface areas is:

a. 4 : 3 b. 16 : 9 c. 9 : 16 d. 2 : 3

Q 14. The ratio of lateral surface area to the total surface area of a cylinder with base diameter 1.6 m height 20 cm is:

a. 1 : 5 b. 5 : 1 c. 3 : 2 d. 2 : 3

Q 15. The radii of two cylinders are in the ratio 3 : 5. If their heights are in the ratio 2 : 3, then the ratio of their curved surface areas is:

a. 2 : 5 b. 5 : 2 c. 3 : 4 d. 4 : 3

Q 16. A solid ball is exactly fitted inside the cubical box of side 2 cm. The volume of the ball is:

a. $\frac{16}{3}\pi\, cm^3$ b. $\frac{4}{3}\pi\, cm^3$

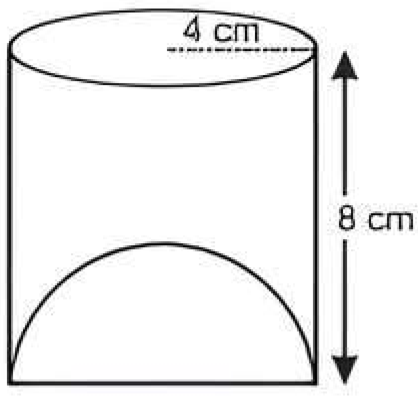
c. $\frac{33}{2}\pi\, cm^3$ d. None of these

Q 17. A solid cylinder of radius R and height H is placed other cylinder of same height and radius. The total surface area of the shape so formed is:

a. $2\pi R(R + 3H)$ b. $2\pi R(R + 2H)$

c. $2\pi R(R - 2H)$ d. None of these

Q 18. The capacity of a cylindrical vessel with a hemispherical portion raised upward at the bottom as shown in the figure is:



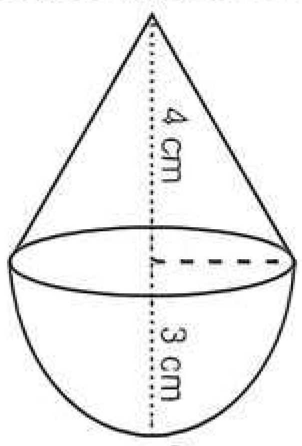
a. $\frac{256}{3}\pi\, cm^3$

b. $\frac{250}{3}\pi\, cm^3$

c. $\frac{245}{3}\pi\, cm^3$

d. $256\pi\, cm^3$

Q 19. The surface area of the following figure is:



a. $20\pi\, cm^2$ b. $33\pi\, cm^2$ c. $18\pi\, cm^2$ d. $17\pi\, cm^2$

Assertion & Reason Type Questions

Directions (Q. Nos. 20-25): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

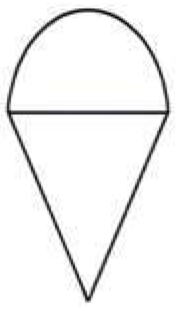
Q 20. Assertion (A): Suppose two equal cubes of edge 4 cm are joined together then the surface area of resulting cuboid is 160 cm^2.

Reason (R): We combined two equal cubes of edge a cm, then the length of the resulting cuboid will be $2a$ cm.

Q 21. Assertion (A): The radii of two cones are in the ratio 2 : 3 and their volumes in the ratio 1 : 3. Then the ratio of their heights is 3 : 4.

Reason (R): Volume of a cone can be determined by the formula, $V = \frac{1}{3}\pi r^2 h$.

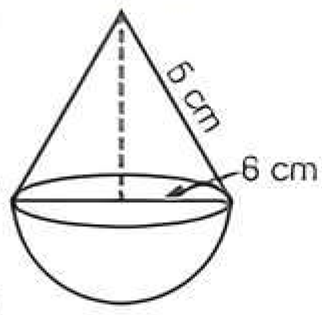
Q 22. Assertion (A): Total surface area of the toy is the sum of the curved surface area of the hemisphere and the curved surface area of the cone.



Reason (R): Toy is obtained by fixing the plane surfaces of the hemisphere and cone together.

[CBSE SQP 2023-24]

Q 23. Assertion (A): The surface area of the combined figure is 320.28 cm^2. [Use $\pi = 3.14$]



Reason (R): The surface area of combined figure is the difference of curved surface areas of individual figures.

Q 24. Assertion (A): If the radius of a cone is halved and volume is not changed, then height remains same.

Reason (R): If the radius of a cone is halved and volume is not changed then height must become four times of the original height.

Q 25. Assertion (A): If the volumes of two spheres are in the ratio 216 : 125. Then their surface areas are in the ratio 6 : 5.

Reason (R): Volume of the sphere $= \frac{4}{3}\pi r^3$ and its surface area $= 4\pi r^2$.

Fill in the Blanks Type Questions

Q 26. The maximum length of rod that can be placed in the cuboid box is equal to its ....

Q 27. If a sphere and a cube have equal surface areas, then the ratio of the volume of the sphere to that of the cube is ....

Q 28. The surface area of a sphere whose volume is 4851 cubic metres is ....

Q 29. If the areas of three adjacent faces of cuboid are x, y, z respectively, then the volume of the cuboid is ....

Q 30. If R and r be the external and internal radius of a hemispherical shell, then the capacity of the hemispherical shell is .... square units.

True/False Type Questions

Q 31. The volume of the material in a hollow body is equal to the difference between the external volume and internal volume.

Q 32. The maximum volume of a cone that can be carved out of a solid hemisphere of radius r is $\frac{1}{2}\pi r^2$.

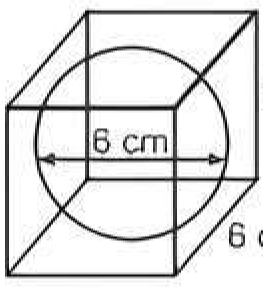
Q 33. The ratio between the volumes of two spheres is 8 : 27, the ratio between their surface areas is 4 : 9.

Q 34. A cube is a special type of cuboid.

Q 35. If we double the radius of a hemisphere, its surface area will also be doubled.

Solutions

1. (c) two cones and one cylinder
2. (a) a sphere and a cylinder
3. (b) Given, edge of a cube is 6 cm.
∴ The maximum diameter of sphere that can be carved out from the cube is 6 cm.



4. (a) Let r and h be the same radii and same height of cylinder and a cone.
∴ Volume of cylinder : Volume of cone

$$= \pi r^2 h : \frac{1}{3}\pi r^2 h = 1 : \frac{1}{3} = 3 : 1$$

5. (a) Given, dimensions of a room are $l = 12$ m, $b = 9$ m and $h = 8$ m.

TR!CK

The length of the longest pole that can be put in a room is $\sqrt{l^2 + b^2 + h^2}$ units.

∴ The length of longest pole

$$= \sqrt{(12)^2 + (9)^2 + (8)^2}$$

$$= \sqrt{144 + 81 + 64} = \sqrt{289} = 17 \text{ m}$$

6. (c) According to the given condition,

$$l + b + h = 6\sqrt{3} \text{ cm} \quad \dots(1)$$

and length of its diagonal, $\sqrt{l^2 + b^2 + h^2} = 2\sqrt{3}$

$\Rightarrow \quad l^2 + b^2 + h^2 = 12$

$\Rightarrow \quad (l + b + h)^2 - 2(lb + bh + hl) = 12$

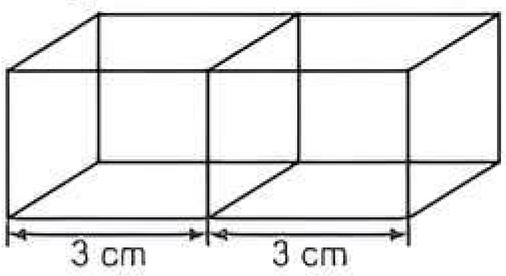
$\Rightarrow \quad (6\sqrt{3})^2 - 12 = 2(lb + bh + hl)$ [from eq. (1)]

$\Rightarrow \quad 2(lb + bh + hl) = 108 - 12 = 96$

∴ Total surface area of the cuboid

$$2(lb + bh + hl) = 96 \text{ cm}^2$$

7. (a) Given, edge of a cube is 3 cm.



TiP

When we combine two cubes, then length of new cuboid is twice the length of cube's edge, but width and height remains same.

Here, length $l = 2 \times 3 = 6$ cm, $b = 3$ cm and $h = 3$ cm.
∴ The surface area of the resulting cuboid

$$= 2(lb + bh + hl)$$
$$= 2(6 \times 3 + 3 \times 3 + 3 \times 6)$$
$$= 2(18 + 9 + 18)$$
$$= 2(45) = 90 \text{ cm}^2$$

8. (d) Let 'r' be the radius of solid hemisphere.
∴ Its diameter $(d) = 2 \times r$

$\Rightarrow \quad r = \frac{d}{2}$

So, total surface area of solid hemisphere
= CSA of hemisphere + Area of circular base

$$= 2\pi r^2 + \pi r^2 = 3\pi r^2$$

$$= 3\pi\left(\frac{d}{2}\right)^2 = \frac{3}{4}\pi d^2$$

9. (a) If two solid hemispheres of same base radius 'r' are joined together along their bases, then new solid becomes a solid sphere.
CSA of new solid (sphere)
= 2 × CSA of a hemisphere of same base radius 'r'

$$= 2 \times 2\pi r^2 = 4\pi r^2$$

10. (b) Given, h and r be the height and radius of cylinder.
If we cut the cylinder from the middle part, then height of remaining cylinder will be $\frac{h}{2}$.
∴ The volume of the remaining cylinder

$$= \pi r^2\left(\frac{h}{2}\right) = \frac{\pi r^2 h}{2}.$$

11. (d) Given, base area of the cone = 156 cm^2
and height of the vertical cone (h) = 8 cm
∴ Volume of the right circular cone

$$= \frac{1}{3} \times \text{base area} \times \text{height}$$

$$= \frac{1}{3} \times 156 \times 8 = 52 \times 8 = 416 \text{ cm}^2$$

12. (b) The maximum height of cone
= Height of the cylinder
= 15 cm

13. (b) Let r_1 and r_2 be the radius of two spheres. Then

$$\frac{\text{Volume of first sphere}}{\text{Volume of second sphere}} = \frac{64}{27}$$

$$\Rightarrow \quad \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{4}{3}\right)^3$$

$$\Rightarrow \quad \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{4}{3}\right)^3$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{4}{3}$$

$\therefore$ Ratio of two surface areas $= \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$

$$= \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

14. (a) Given, base diameter of a cylinder, $d = 1.6$ m and height, $h = 20$ cm.

$\therefore$ Radius of cylinder, $r = \frac{d}{2} = \frac{1.6}{2}$ m

$= 0.8$ m $= 80$ cm

So, the ratio of lateral surface area to the total surface area of a cylinder $= \frac{2\pi rh}{2\pi r(h+r)} = \frac{h}{(h+r)}$

$$= \frac{20}{(20+80)} = \frac{20}{100} = \frac{1}{5}$$

$\therefore$ Required ratio is 1 : 5.

15. (a) Given, radii and heights of two cylinders are in ratio $\frac{r_1}{r_2} = \frac{3}{5}$ and $\frac{h_1}{h_2} = \frac{2}{3}$

$\therefore$ The ratio of two curved surface areas of two cylinders $= \frac{2\pi r_1 h_1}{2\pi r_2 h_2} = \left(\frac{r_1}{r_2}\right)\left(\frac{h_1}{h_2}\right) = \frac{3}{5} \times \frac{2}{3} = \frac{2}{5}$

$\therefore$ Required ratio is 2 : 5.

16. (b)

TiP

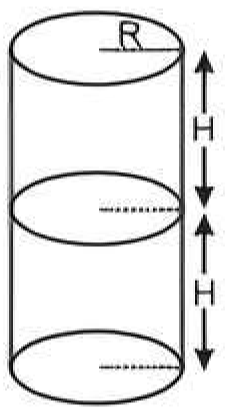
If solid ball is exactly fitted inside the cubical box of side 'a' unit. Then the diameter of the solid ball will be 'a' unit.

Here, diameter of the ball is 2 cm. So, its radius will be 1 cm.

$\therefore$ The volume of the ball =— π $= \frac{4}{3} \times \pi(1)^3$

$$= \frac{4}{3}\pi \text{ cm}^3$$

17. (b) From the figure, it is clear that total height of a cylinder is $2H$ and radius of a cylinder is R.



$\therefore$ Total surface area of cylinder

$= 2\pi R(R + 2H)$

18. (a)

TR!CKS

- *The capacity of cylindrical vessel* $= \pi r^2 h$ *cubic units.*
- *The capacity of hemisphere* $= \frac{2}{3}\pi r^3$ *cubic units.*

Given, radius and height of a cylinder are $r = 4$ cm and $h = 8$ cm.

$\therefore$ The capacity of given vessel = Volume of cylinder − Volume of hemisphere

$$= \pi r^2 h - \frac{2}{3}\pi r^3$$

$$= \frac{\pi r^2}{3}(3h - 2r)$$

$$= \frac{\pi}{3}(4)^2\,(3 \times 8 - 2 \times 4)$$

$$= \frac{16\pi}{3}(24 - 8)$$

$$= \frac{16\pi}{3} \times 16 = \frac{256\pi}{3} \text{ cm}^3$$

19. (b) Given radius of cone and hemisphere is $r = 3$ cm and height of cone is $h = 4$ cm.

TiP

The slant height of a cone is $l = \sqrt{r^2 + h^2}$.

Now, slant height of a cone, $l = \sqrt{r^2 + h^2}$

$$= \sqrt{(3)^2 + (4)^2}$$

$$= \sqrt{9 + 16} = \sqrt{25}$$

$$= 5 \text{ cm}$$

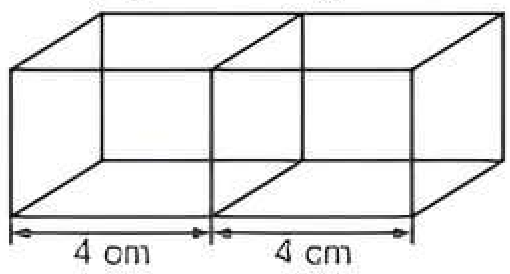
$\therefore$ The surface area of given figure

= Curved surface area of cone + curved surface area of hemisphere

$$= \pi r l + 2\pi r^2 = \pi r\,(l + 2r)$$

$$= \pi \times 3\,(5 + 2 \times 3) = 3\pi\,(5 + 6) = 33\pi \text{ cm}^2$$

20. (a) **Assertion (A):** Given edge of a cube is 4 cm.



TiP

When we combine two cubes, then length of new cuboid is twice the original edge, but width and height remains same.

Here, length, $l = 2 \times 4 = 8$ cm.

width, $b = 4$ cm

and height, $h = 4$ cm

$\therefore$ The surface area of resulting cuboid

$$= 2\,(lb + bh + hl)$$

$$= 2\,(8 \times 4 + 4 \times 4 + 4 \times 8)$$

$$= 2\,(32 + 16 + 32) = 2 \times 80 = 160 \text{ cm}^2$$

So, Assertion (A) is correct.

Reason (R): It is true to say that length of the combined cubes is $2a$ cm.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

21. (a) **Assertion (A):** Let r_1 and r_2 be the radii of two cones and h_1 and h_2 be the heights of two cones. Then

$$\frac{r_1}{r_2} = \frac{2}{3} \quad ...(1)$$

Now, ratio of two volumes is $\frac{V_1}{V_2} = \frac{1}{3}$

$$\Rightarrow \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{1}{3} \Rightarrow \left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right) = \frac{1}{3}$$

$$\Rightarrow \left(\frac{2}{3}\right)^2 \times \left(\frac{h_1}{h_2}\right) = \frac{1}{3} \quad (\because \text{ from eq. (1)})$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{1}{3} \times \frac{9}{4} = \frac{3}{4}$$

So, Assertion (A) is true.

Reason (R): It is true that volume of any cone can be determined by the formula

$$V = \frac{1}{3}\pi r^2 h$$

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

22. (a) **Assertion (A):** We know that, the surface area of combined figure is the sum of all the curved surface of the individual figures.

TSA of the toy = CSA of hemisphere + CSA of the cone.

So, Assertion (A) is true.

Reason (R): It is also a true statement because any combined figure is obtained by fixing the plane surfaces of individual figures.

So, Reason (R) is true.

Hence, both Assertion (A) and Reason (R) is true and Reason (R) is the correct explanation of Assertion (A).

23. (c) **Assertion (A):** Given, slant height of cone, $l = 5$ cm, radius of cone = radius of hemisphere $r = 6$ cm.

∴ Surface area of the combined figure

$= $ CSA of cone + CSA of hemisphere

$= \pi rl + 2\pi r^2$

$= 3.14 \times 6 \times 5 + 2 \times 3.14 \times (6)^2$

$= 94.2 + 226.08$

$= 320.28 \text{ cm}^2$

So, Assertion (A) is true.

Reason (R): It is a false statement. The right statement is that the surface area of the combined figure is the sum of curved surface areas of individual figures.

Hence, Assertion (A) is true but Reason (R) is false.

24. (d) **Assertion (A):** If the radius of a cone is halved and volume is not changed, then height is not remain same.

So, Assertion (A) is false.

Reason (R): Let V_1 and V_2 be the volume of two cones. Then

$$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$\frac{V_1}{V_2} = \frac{r^2 h_1}{\left(\frac{r}{2}\right)^2 h_2}$$

But V_1 and V_2 are equal.

$$\therefore \quad r^2 h_1 = \left(\frac{r}{2}\right)^2 h_2$$

$$\Rightarrow \quad h_1 = \frac{1}{4} h_2$$

$$\Rightarrow \quad 4h_1 = h_2$$

So, Reason (R) is true.

Hence, Assertion (A) is false but Reason (R) is true.

25. (b) **Assertion (A):** Let V_1 and V_2 be the volumes of two spheres. Also, let r_1 and r_2 be the radius of two spheres respectively. Then

$$\frac{V_1}{V_2} = \frac{216}{125}$$

$$\Rightarrow \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{6}{5}\right)^3$$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{6}{5}\right)^3$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{6}{5}$$

∴ The ratio of surface areas of two spheres is

$$\frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{6}{5}\right)^2 = \frac{36}{25}$$

Reason (R): It is also a true statement.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

26. diagonal

27. Let r be the radius of sphere and 'a' be the side of a cube.

Given that,

Surface area of sphere = Surface area of a cube

$$\therefore \quad 4\pi r^2 = 6(a)^2$$

$$\Rightarrow \quad \frac{r^2}{a^2} = \frac{6}{4\pi}$$

$$\Rightarrow \quad \frac{r}{a} = \frac{\sqrt{6}}{2\sqrt{\pi}}$$

Now, $\dfrac{\text{Volume of sphere}}{\text{Volume of cube}} = \dfrac{\frac{4}{3}\pi r^3}{(a)^3}$

$= \dfrac{4}{3}\pi \times \left(\dfrac{r}{a}\right)^2 \left(\dfrac{r}{a}\right)$

$= \dfrac{4}{3}\pi \times \dfrac{6}{4\pi} \times \left(\dfrac{\sqrt{6}}{2\sqrt{\pi}}\right) = \sqrt{\dfrac{6}{\pi}}$

$= \sqrt{6} : \sqrt{\pi}$

28. Given, volume of a sphere, $V = 4851\ m^3$

Let r be the radius of sphere.

$\because \quad V = 4851\ m^3$

$\Rightarrow \quad \dfrac{4}{3}\pi r^3 = 4851 \Rightarrow \dfrac{4}{3} \times \dfrac{22}{7} \times r^3 = 4851$

$\Rightarrow \quad r^3 = \dfrac{4851 \times 3 \times 7}{4 \times 22} = \dfrac{441 \times 21}{8}$

$\Rightarrow \quad r^3 = \left(\dfrac{21}{2}\right)^3 \Rightarrow r = \dfrac{21}{2}$ cm

$\therefore$ Surface area of sphere $= 4\pi r^2 = 4 \times \dfrac{22}{7} \times \left(\dfrac{21}{2}\right)^2$

$= 1386\ cm^2$

29. Let l, b and h be the length, breadth and height of a cuboid.

Then, $\quad x = lb,\ y = bh$ and $z = hl$

Now $\quad xyz = lb \times bh \times hl$

$xyz = l^2b^2h^2$

$\Rightarrow \quad lbh = \sqrt{xyz}$

So, the volume of cuboid $= lbh = \sqrt{xyz}$.

30. $\dfrac{2}{3}\pi (R^3 - r^3)$ cubic units.

31. True

32. To find out the maximum volume of cone that can be carved out from hemisphere, the radius and height of cone will be r.

$\therefore$ The maximum volume of cone $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3}\pi r^2 \times r = \dfrac{\pi r^3}{3}$

Hence, given statement is false.

33. Let r_1 and r_2 be the radii of two spheres. Then ratio of volume of spheres is

$\dfrac{V_1}{V_2} = \dfrac{8}{27}$

$\Rightarrow \quad \dfrac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \dfrac{8}{27} \quad \Rightarrow \quad \left(\dfrac{r_1}{r_2}\right)^3 = \dfrac{8}{27}$

$\Rightarrow \quad \left(\dfrac{r_1}{r_2}\right)^3 = \left(\dfrac{2}{3}\right)^3 \quad \Rightarrow \quad \dfrac{r_1}{r_2} = \dfrac{2}{3} \qquad ...(1)$

$\therefore$ The ratio between their surface areas is

$\dfrac{S_1}{S_2} = \dfrac{4\pi r_1^2}{4\pi r_2^2} = \left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9}.$

Hence, the given statement is true.

34. True, because in a cube, length, breadth and height are all equal.

35. Let original radius of sphere be r unit. Then surface area of hemisphere

$S_1 = 2\pi r^2$

If $\quad r = 2r$, then

$S_2 = 2\pi (2r)^2$

$= 8\pi r^2$

$S_2 = 4(2\pi r^2)$

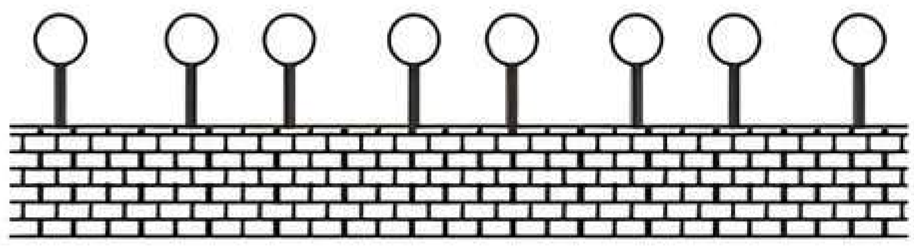
$S_2 = 4S_1$

So, given statement is false.

Case Study Based Questions

Case Study 1

The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in figure. Eight such spheres are used for this purpose and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and it is to be painted black.



Based on the above information, solve the following questions:

Q 1. The surface area of a sphere is:

a. $1386\ cm^2$ b. $1496\ cm^2$
c. $1290\ cm^2$ d. $1426\ cm^2$

Q 2. The cost of silver paint of 8 spheres at the rate of 25 paise per cm^2 is:

a. ₹ 1257.68 b. ₹ 2757.86
c. ₹ 3128.63 d. ₹ 3356.23

Q 3. The surface area of a cylindrical part is:

a. $22\ cm^2$ b. $33\ cm^2$ c. $44\ cm^2$ d. $66\ cm^2$

Q 4. The cost of black paint of 8 cylinders at the rate of 5 paise per cm^2 is:

a. ₹ 26.40 b. ₹ 16.90 c. ₹ 19.30 d. ₹ 23.30

Q 5. The total cost of painting spheres with their support is:

a. ₹ 1239.62 b. ₹ 2784.26
c. ₹ 3134.16 d. ₹ 4196

Solutions

1. Radius of a sphere $(r) = \dfrac{21}{2}$ cm $= 10.5$ cm

$\therefore$ Surface area of a sphere $= 4\pi r^2$

$= 4 \times \dfrac{22}{7} \times 10.5 \times 10.5\ cm^2 = 1386\ cm^2$

So, option (a) is correct.

2. Radius of the cylinder $(R) = 1.5$ cm
Height of the cylinder $(h) = 7$ cm
∴ Area of the base of the cylinder (support)

$$= \pi R^2 = \pi \times (1.5)^2$$

$$= \frac{22}{7} \times 1.5 \times 1.5 \text{ cm}^2 = 7.07 \text{ cm}^2$$

Now, area of a sphere to paint silver
= Surface area of a sphere
– Area of the base of the cylinder

$$= (1386 - 7.07) \text{ cm}^2 = 1378.93 \text{ cm}^2$$

Area of sphere to be painted silver

$$= 8 \times 1378.93 \text{ cm}^2$$

∴ Cost of painting the spheres

$$= ₹ \frac{8 \times 1378.93 \times 25}{100}$$

$$= ₹ 2757.86$$

So, option (b) is correct.

3. Now, curved surface area of a cylindrical part (support)

$$= 2\pi Rh = 2 \times \frac{22}{7} \times 1.5 \times 7 \text{ cm}^2 = 66 \text{ cm}^2$$

So, option (d) is correct.

4. Curved surface area of 8 cylindrical supports

$$= 8 \times 2\pi Rh$$

$$= 8 \times 2 \times \frac{22}{7} \times 1.5 \times 7 \text{ cm}^2$$

∴ Cost of painting the supports

$$= ₹ 8 \times 2 \times \frac{22}{7} \times 1.5 \times 7 \times \frac{5}{100} = ₹ 26.40$$

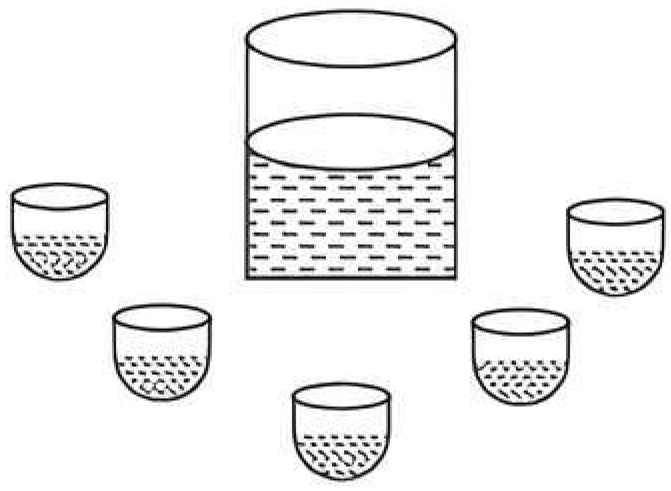
So, option (a) is correct.

5. Total cost required for painting
= Total cost of 8 silver paint spheres
+ Total cost of 8 black paint cylinder
= ₹ (2757.86 + 26.40) = ₹ 2784.26

So, option (b) is correct.

Case Study 2

In some Muslim countries, eating in public during day-light hours in Ramadan is crime. The sale of alcohol becomes prohibited during Ramadan and alcohol is completely restricted in Ramadan Mela. At a Ramadan Mela, a stall keeper in one of the food stalls has a large cylindrical vessel of base radius 15 cm filled up to a height of 32 cm with orange juice. The juice is filled in small glasses, each small glass consist of a 6 cm long cylindrical portion attached to a hemisphere of radius 3 cm and sold for ₹ 15 each.



Based on the above information, solve the following questions:

Q 1. The volume of juice in the vessel is:
a. $1200 \pi \text{ cm}^3$
b. $4200 \pi \text{ cm}^3$
c. $5200 \pi \text{ cm}^3$
d. $7200 \pi \text{ cm}^3$

Q 2. The capacity of each small glass is:
a. $72 \pi \text{ cm}^3$
b. $42 \pi \text{ cm}^3$
c. $32 \pi \text{ cm}^3$
d. $64 \pi \text{ cm}^3$

Q 3. Number of glasses of juice that are sold:
a. 10
b. 50
c. 100
d. 90

Q 4. How much money does the stall keeper receive by selling the juice completely?
a. ₹ 1500
b. ₹ 750
c. ₹ 1250
d. ₹ 1750

Q 5. If 1/4 part of juice fall initially by stall keeper and then sold remaining juice for ₹ 25 each. How much money does the stall keeper receive by selling the remaining juice completely?
a. ₹ 550
b. ₹ 1875
c. ₹ 650
d. ₹ 750

Solutions

1. Given that
Radius of the cylindrical vessel $(R) = 15$ cm
and height of the cylindrical vessel $(H) = 32$ cm
∴ The volume of juice in the vessel
= Volume of the cylindrical vessel

$$= \pi R^2 H$$

$$= \pi \times 15 \times 15 \times 32 = 7200 \pi \text{ cm}^3$$

So, option (d) is correct.

2. Given that
Height of the small glass $(h) = 6$ cm
and radius of the small glass (r)
= Radius of cylinder = Radius of hemisphere
= 3 cm
∴ The capacity of juice in each glass can hold
= Volume of each small glass
= Volume of small cylinder
+ Volume of small hemisphere

$$= \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{\pi r^2}{3}(3h + 2r)$$

$$= \frac{\pi(3)^2}{3}(3 \times 6 + 2 \times 3)$$

$$= 3\pi (18 + 6) = 72\pi \text{ cm}^3$$

So, option (a) is correct.

3. The number of glasses of juice that are sold

$$= \frac{\text{Volume of the vessel}}{\text{Volume of each glass}}$$

$$= \frac{7200\pi}{72\pi} = 100$$

So, option (c) is correct.

4. Amount received by the stall keeper
= ₹ 15 × 100 = ₹ 1500

So, option (a) is correct.

5. $\because$ The volume of juice in the vessel $= 7200\pi\ cm^3$

$\therefore$ Volume of $\frac{1}{4}$ part of juice

$$= \frac{1}{4} \times 7200\pi = 1800\pi\ cm^3$$

$\therefore$ Volume of remaining juice

$$= 7200\pi - 1800\pi = 5400\pi\ cm^3$$

$\therefore$ The number of glasses of juice that are sold

$$= \frac{5400\pi}{72\pi} = 75$$

$\therefore$ Amount received by the stall keeper

$$= ₹\ 25 \times 75 = ₹\ 1875$$

So, option (b) is correct.

Case Study 3

On diwali festival, a big company decided to gift his employees an electric kettle which was in a shape of cylinder and gift wrapped in the cubical box. The dimension of box is 20 cm × 15 cm × 30 cm and the radius and height of electrical kettle are 14 cm and 25 cm.


Based on the above information, solve the following questions:

Q 1. Find the volume of the box.

Q 2. Find the area of the wrapping sheet that covers the box exactly.

Or

Find the total surface area of an electric kettle.

Q 3. Find the maximum length of rod that can be kept in the box.

Solutions

1. Given, dimension of a box is $l = 20$ cm, $b = 15$ cm and $h = 30$ cm

The volume of the box $= lbh$

$$= 20 \times 15 \times 30 = 9000\ cm^3.$$

Hence, volume of the box is $9000\ cm^3$.

2. The area of the wrapping sheet that covers the box is equal to the surface area of the box.

$\therefore$ Surface area of the box $= 2(lb + bh + hl)$

$$= 2(20 \times 15 + 15 \times 30 + 30 \times 20)$$
$$= 2(300 + 450 + 600)$$
$$= 2(1350) = 2700\ cm^2.$$

Hence, the area of the wrapping sheet that covers the box exactly is $2700\ cm^2$.

Or

Given, radius and height of an electric kettle are $r = 14$ cm and $h = 25$ cm.

$\therefore$ The total surface area of an electric kettle

$$= \text{total surface area of cylinder}$$
$$= 2\pi r(h + r)$$
$$= 2 \times 3.14 \times 14\ (25 + 14)$$
$$= 87.92 \times 39 = 3428.88\ cm^2$$

Hence, surface area of an electric kettle is $3428.88\ cm^2$.

3.

TiP

The maximum length of rod that can be kept in the box, is equal to the length of diagonal of a cuboid.

$\therefore$ The maximum length of rod

$$= \text{length of diagonal of a cuboid}$$
$$= \sqrt{l^2 + b^2 + h^2} = \sqrt{(20)^2 + (15)^2 + (30)^2}$$
$$= \sqrt{400 + 225 + 900} = \sqrt{1525}$$
$$= 5\sqrt{61}\ cm$$

Hence, a maximum length that can be kept in the box is $5\sqrt{61}$ cm.

Case Study 4

In a coffee shop, coffee is served in two types of cups. One is cylindrical in shape with diameter 7 cm and height 14 cm and the other is hemispherical with diameter 21 cm.


Based on the above information, solve the following questions: [CBSE 2023]

Q 1. Find the area of the base of the cylindrical cup.

Q 2. What is the capacity of the hemispherical cup?

Or

Find the capacity of the cylindrical cup.

Q 3. What is the curved surface area of the cylindrical cup?

Solutions

1. Let r and h be the radius and height of the cylindrical cup respectively.

Given, diameter of the base $= 7$ cm

$\therefore$ Its radius $(r) = \frac{7}{2}$ cm

So, base area of the cylindrical cup $= \pi r^2$

$$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{22}{7} \times \frac{49}{4} = \frac{77}{2}$$
$$= 38.5\ cm^2$$

2. Let R be the radius of the hemispherical cup.

$\therefore$ Given, diameter of hemispherical cup = 21 cm

Its radius $(R) = \frac{21}{2}$ cm

So, capacity of the hemispherical cup

$$= \frac{2}{3}\pi R^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^3 = \frac{11 \times 21 \times 21}{2}$$

$$= 2425.5 \text{ cm}^3.$$

Or

Given, height of the cylindrical cup $(h) = 14$ cm

and its radius $(r) = \frac{7}{2}$ cm

Capacity of the cylindrical cup

$$= \pi r^2 h = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 14$$

$$= 11 \times 7 \times 7 = 539 \text{ cm}^3$$

3. In cylindrical cup,

Given, radius $(r) = \frac{7}{2}$ cm and height $(h) = 14$ cm

$\therefore$ Curved surface area of the cylindrical cup

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 14 = 308 \text{ cm}^2$$

Case Study 5

A 'circus' is a company of performers who put on shows of acrobats, clowns etc. to entertain people started around 250 years back, in open fields, now generally performed in tents.

One such 'Circus Tent' is shown below:

The tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of cylindrical part are 9 m and 30 m respectively and height of conical part is 8 m with same diameter as that of the cylindrical part.

Based on the above information, solve the following questions: [CBSE 2022 Term-II]

Q 1. The area of the canvas used in making the tent.

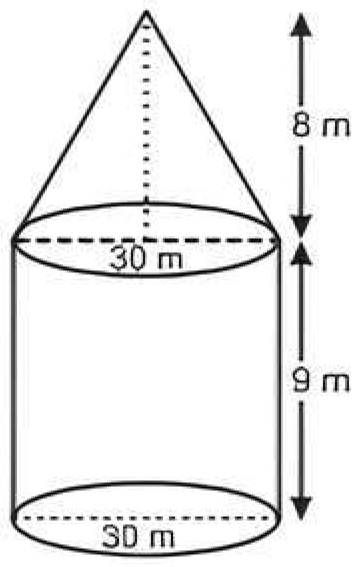
Q 2. The cost of the canvas bought for the tent as the rate ₹ 200 per sq. m, if 30 sq. m canvas was wasted during stitching.

Solutions

1. Given, diameter of the cylindrical part and conical part are same.

$\therefore$ d = diameter of cylinder = diameter of cone = 30 m

$\Rightarrow r$ = radius of cylinder = radius of cone $= \frac{30}{2} = 15$ m



Also height of the cylindrical part $(h_2) = 9$ m

and height of the conical part $(h_1) = 8$ m

The area of the canvas used in making the tent = curved surface area of cylinder

$$= \pi rl + 2\pi rh_2$$

$$= \pi r\sqrt{r^2 + h_1^2} + 2\pi rh_2$$

$$= \frac{22}{7} \times 15 \times \sqrt{(15)^2 + (8)^2} + 2 \times \frac{22}{7} \times 15 \times 9$$

$$= \frac{22}{7} \times 15 \times \sqrt{225 + 64} + \frac{5940}{7}$$

$$= \frac{330}{7} \times \sqrt{289} + \frac{5940}{7} = \frac{330 \times 17}{7} + \frac{5940}{7}$$

$$= \frac{5610}{7} + \frac{5940}{7} = \frac{11550}{7}$$

$$= 1650 \text{ m}^2$$

Hence, the area of the canvas used in making the tent is 1650 m^2.

2. Since, the 30 sq. m of canvas wasted during stitching, Therefore total area of canvas using in making the tent = 1650 + 30

$$= 1680 \text{ m}^2$$

$\because$ The cost of 1 m^2 canvas bought is ₹ 200

$\therefore$ The cost of 1680 m^2 canvas bought is ₹ 1,680 × 200

= ₹ 3,36,000

Very Short Answer Type Questions

Q 1. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere? [CBSE 2017]

Q 2. If two cubes of edge 2 cm each are joined end to end, find the surface area of the resulting cuboid. [V. Imp.]

Q 3. A ball is exactly fit inside the cubical box of side '*a*' unit. What is the volume of the ball? [V. Imp.]

Q 4. Two cubes have their volumes in the ratio 8 : 27. Find the ratio of their surface areas.

Q 5. A cone of radius 5 cm and slant height 13 cm, then find the height of cone.

Short Answer Type-I Questions

Q 1. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part. [CBSE 2020]

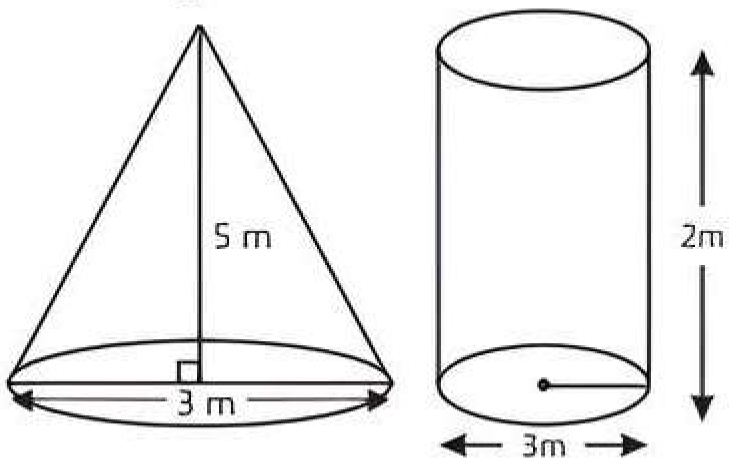
Q 2. Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape formed.

Q 3. What will be the approximate volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm?

Q 4. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder. $\left[\text{Use } \pi = \frac{22}{7}\right]$. [CBSE 2020]

Q 5. From a solid right circular cylinder of height 14 cm and base radius 6 cm, a right circular cone of same height and same base radius is removed. Find the volume of the remaining solid. [CBSE 2020]

Q 6. Two types of water tankers are available in a shop at the same rate.
First one is in a conical form of diameter 3 m and height 5 m.
Second one is in the form of a cylinder of diameter 3 m and height 2 m.



Out of the two, which tanker capacity (in litres) is more and how much?

Q 7. A factory manufactures 120000 pencils daily. The pencils are cylindrical in shape each of length 25cm and circumference of base as 1.5 cm. Determine the cost of colouring the curved surfaces of the pencils manufactured in one day at ₹ 0.06 per dm^2.

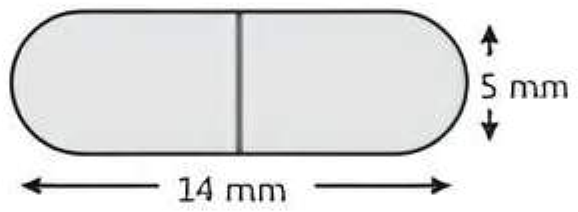
Q 8. The curved surface area of a right circular cone is 12320 cm^2. If the radius of its base is 56 cm, then find its height. [CBSE SQP 2022 Term-II]

Short Answer Type-II Questions

Q 1. A solid is in the shape of a right-circular cone surmounted on a hemisphere, the radius of each of them being 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid. [CBSE 2023]

Q 2. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see the given figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area. $\left[\text{Use } \pi = \frac{22}{7}\right]$

[NCERT EXERCISE; V. Imp.]



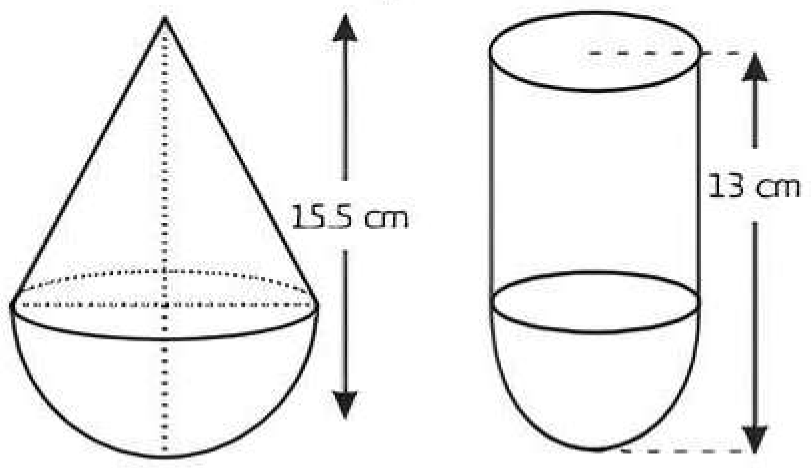
Q 3. From a solid cylinder of height 20 cm and diameter 12 cm, a conical cavity of height 8 cm and radius 6 cm is hollowed out. Find the total surface area of the remaining solid. [CBSE 2023]

Q 4. The cost of painting the total outside surface of a closed cylindrical oil tank at 60 paise per sq. m is ₹ 237.60 and the height of the tank is 6 times the radius of the base of the tank. Find the radius and height of the tank. $\left[\text{Use } \pi = \frac{22}{7}\right]$ [CBSE 2015]

Q 5. A wire of diameter 3 mm is wound about a cylinder whose height is 12 cm and radius 5 cm so as to cover the curved surface of the cylinder completely. Find the length of the wire. [CBSE 2017]

Q 6. A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap? [NCERT EXEMPLAR; CBSE 2018]

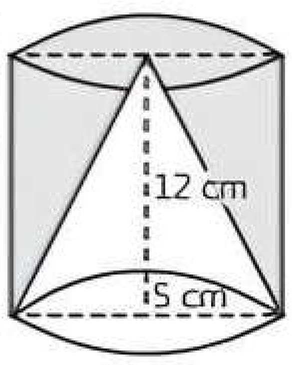
Q 7. Aditi went to a painter to get her two wooden toys painted.
Toy 1 is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius.



Toy 2 is in the form of a hemispherical bowl of diameter 7 cm mounted by a hollow cylinder.
These toys are painted from inside.
Out of the two, whose cost of painting at ₹ 1.50 per cm^2 is more and by how much?

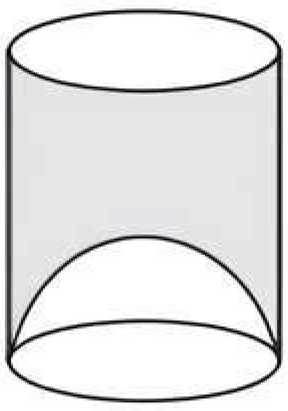
Q 8. The sum of the radius of base and height of a solid right circular cylinder is 37cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. $\left[\text{Use } \pi = \frac{22}{7}\right]$ [CBSE 2016]

Q 9. From a solid right circular cylinder with height 12 cm and radius of the base 5 cm, a right circular cone of the same height and the same base radius is removed. Find the volume and total surface area of the remaining solid. [Use $\pi = 3.14$]



[CBSE 2015]

Q 10. A Juice seller was serving juice to his customers using glasses as shown in figure. The inner diameter of the cylindrical glass was 5 cm but bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent and actual capacity of the glass. [Use $\pi = 3.14$].



[NCERT EXERCISE; CBSE 2019]

Long Answer Type Questions

Q 1. A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of the hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}$ cm^3. Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of ₹ 10 per cm^2. *[CBSE 2022 Term-II]*

Q 2. From a cuboidal solid metallic block of dimensions $15 cm \times 10 cm \times 5 cm$, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block in cuboidal block.

Q 3. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively decided to provide place and the canvas for 1500 tents and share the whole expenditure equally. The lower part of each tent is cylindrical with base radius 2.8 m and height 3.5 m and the upper part is conical with the same base radius, but of height 2.1 m. If the canvas used to make the tents costs ₹ 120 per m^2, find the amount shared by each school to set up the tents. *[CBSE SQP 2022-23]*

Q 4. An iron pole consists of a cylinder of height 240 cm and base diameter 26 cm, which is surmounted by another cylinder of height 66 cm and radius 10 cm. Find the mass of the pole given that 1 cm^3 of iron has approximately 8 g mass. [Take, $\pi = 3.14$]

Q 5. A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of the cylinder. The diameter and height of the cylinder are 6 cm and 12 cm, respectively. If the slant height of the conical portion is 5 cm, then find the total surface area and volume of the rocket. [Use $\pi = 3.14$]

Q 6. A tent is in the shape of a cylinder surmounted by a conical top. If the height and radius of the cylindrical part are 3 m and 14 m respectively and the total height of the tent is 13.5 m, find the area of the canvas required for making the tent, keeping a provision of 26 m^2 of canvas for stitching and wastage. Also, find the cost of the canvas to be purchased at the rate of ₹ 500 per m^2.

[CBSE SQP 2023-24]

Q 7. Water is following at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in pond rise by 21 cm. What should be the speed of water if the rise in water level is to be attained in 1 hour? *[CBSE SQP 2023-24]*

Q 8. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder as shown in the figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article. *[CBSE 2023]*

Q 9. There are two identical cubical boxes of side 7 cm. From the top face of the first cube a hemisphere of diameter equal to the side of the cube is scooped out. This hemisphere is inverted and placed on the top of the second cube's surface to form a dome. Find:

(i) the ratio of the total surface area of the two new solids formed.

(ii) the volume of each new solid formed.

[CBSE SQP 2022-23]

Solutions

Very Short Answer Type Questions

1. Let the radius of the hemisphere be r.

$\therefore$ Volume of hemisphere $= \frac{2}{3}\pi r^3$

and total surface area of solid hemisphere $= 3\pi r^2$

According to the question.

Volume of hemisphere

= Total surface area of hemisphere

$$\Rightarrow \quad \frac{2}{3}\pi r^3 = 3\pi r^2 \quad \Rightarrow \quad 2r^3 = 9r^2$$

$$\Rightarrow \quad r^2\left(r - \frac{9}{2}\right) = 0$$

$$\Rightarrow \quad r = \frac{9}{2} \qquad (\because r \neq 0)$$

So. the diameter of hemisphere = $2r$

$$= 2 \times \frac{9}{2} = 9 \text{ units}$$

COMMON ERR(!)R

Students take the formula of TSA of solid hemisphere as $2\pi r^2$ in haste but it is wrong. Students should understand the formula of TSA of hemisphere, which is

$$2\pi r^2 + \pi r^2 = 3\pi r^2.$$

2. Given, edge of cube = 2 cm

$\therefore$ Length of cuboid formed $(l) = 2 + 2 = 4$ cm

Breadth of cuboid $(b) = 2$ cm

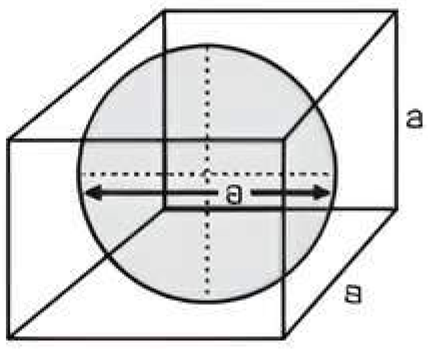
and height of cuboid $(h) = 2$ cm.

$\therefore$ Surface area of the resulting cuboid

$$= 2(lb + bh + hl)$$
$$= 2(4 \times 2 + 2 \times 2 + 2 \times 4)$$
$$= 2(8 + 4 + 8)$$
$$= 2 \times 20 = 40 \text{ cm}^2$$

3. Given,

Diameter of the ball = Edge of the cubical box = 'a' units



$\therefore$ Radius of the ball $(r) = \frac{a}{2}$ units

Hence, volume of the ball $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi\left(\frac{a}{2}\right)^3$$
$$= \frac{\pi a^3}{6} \text{ cubic units}$$

4. Let 'a' and 'A' be the edges of two cubes. Then

$$\frac{\text{Volume of first cube}}{\text{Volume of second cube}} = \frac{8}{27}$$

$$\Rightarrow \quad \frac{a^3}{A^3} = \left(\frac{2}{3}\right)^3$$

$$\Rightarrow \quad \frac{a}{A} = \frac{2}{3}$$

$\therefore$ Ratio of surface areas of two cubes $= \frac{6a^2}{6A^2}$

$$= \left(\frac{a}{A}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9} = 4:9$$

5. Given radius of cone $r = 5$ cm

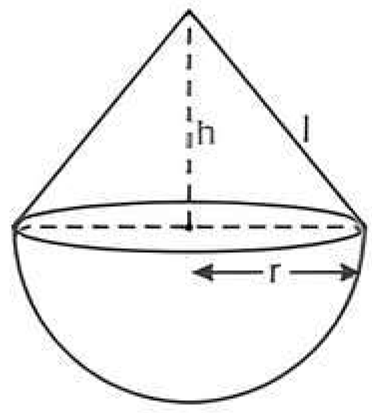
and slant height of cone $(l) = 13$ cm

$$\because \quad h = \sqrt{l^2 - r^2}$$
$$= \sqrt{(13)^2 - (5)^2} = \sqrt{169 - 25}$$
$$= \sqrt{144} = 12 \text{ cm}$$

Hence, height of cone is 12 cm.

Short Answer Type-I Questions

1. Let the height and slant height of the cone are h and l respectively.



According to the question,

Radius of cone = Radius of hemisphere = r

Now, CSA of conical part = CSA of hemispherical part

$$\Rightarrow \quad \pi rl = 2\pi = r^2$$

$$\Rightarrow \quad \sqrt{r^2 + h^2} = 2r \qquad (\because l = \sqrt{h^2 + r^2})$$

$$\Rightarrow \quad r^2 + h^2 = 4r^2 \Rightarrow h^2 = 3r^2$$

$$\Rightarrow \quad \frac{r^2}{h^2} = \frac{1}{3} \Rightarrow \frac{r}{h} = \frac{1}{\sqrt{3}}$$

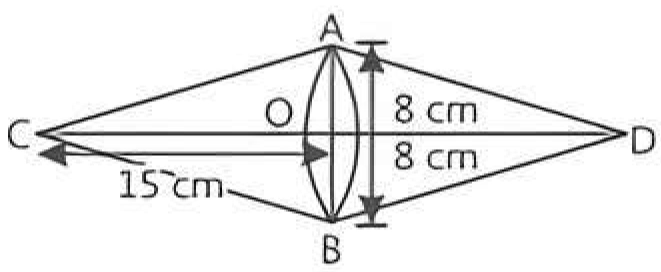
Hence, required ratio is $1:\sqrt{3}$.

2. Given, radius of cone, $(r) = 8$ cm

and height of cone, $(h) = 15$ cm

Now, slant height of cone ABC,

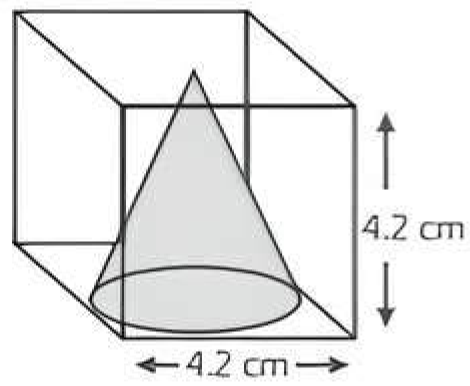
$$l = \sqrt{(OA)^2 + (OC)^2} = \sqrt{(8)^2 + (15)^2}$$
$$= \sqrt{64 + 225} = \sqrt{289} = 17 \text{ cm}$$



Now, surface area of the joined shape

$$= 2 \times \text{Curved surface area of cone ABC}$$
$$= 2\pi rl$$
$$= 2 \times \frac{22}{7} \times 8 \times 17$$
$$= \frac{5984}{7} = 854.86 \text{ cm}^2$$

3. Given, edge of a cube = 4.2 cm



From figure,

Height of cone (h) = Diameter of cone

= Edge of cone = 4.2 cm

$\therefore$ Radius of cone $(r) = \frac{4.2}{2} = 2.1$ cm

$\therefore$ Volume of the largest right circular cone

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 4.2$$
$$= 22 \times 0.3 \times 0.7 \times 4.2 = 19.404 \text{ cm}^3$$

4. Given, height of a right circular cylinder (h) = radius of a right circular cylinder $(r) = x$ (say).

and volume of cylinder $= 25\frac{1}{7} = \frac{176}{7}$ cm^3

$$\therefore \quad \pi r^2 h = \frac{176}{7}$$

$$\Rightarrow \quad \frac{22}{7} \times x^2 \times x = \frac{176}{7}$$

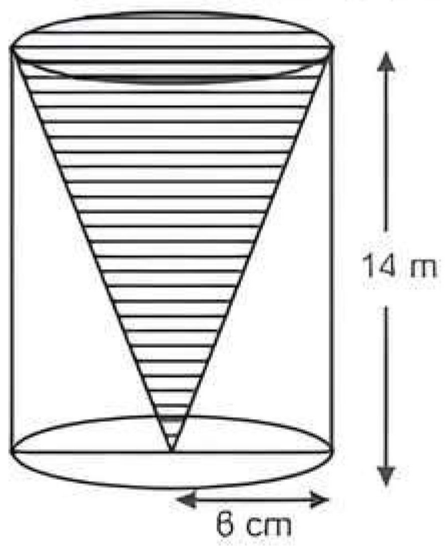
$$\Rightarrow \quad x^3 = 8 = 2^3$$

$$\Rightarrow \quad x = 2$$

Hence, required height is 2 cm.

5. Given that,

height of the cylinder (h) = height of the cone = 14 cm



and base radius of the cylinder = base radius of the cone

$$\Rightarrow \quad (r) = 6 \text{ cm}$$

$\therefore$ The volume of the remaining solid = Volume of cylinder – Volume of cone

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3}\pi \times (6)^2 \times 14 = \frac{2}{3} \times \frac{22}{7} \times 36 \times 14 = 1056 \text{ cm}^3$$

6. Given, radius of cone $(r) = \frac{3}{2}$ m and height of cone $(h) = 5$

$$\therefore \quad \text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 5$$

$$= 11.7857 \text{ m}^3$$

$$= 11785.7 \text{ L} \qquad (\because 1 \text{ m}^3 = 1000 \text{ L})$$

Also, radius of cylinder $(R) = \frac{3}{2}$ m ;

and height of cylinder $(H) = 2$ m

$$\therefore \quad \text{Volume of cylinder} = \pi R^2 H$$

$$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 2$$

$$= 14.1428 \text{ m}^3$$

$$= 14142.8 \text{ L} \ (\because 1 \text{ m}^3 = 1000 \text{ L})$$

Hence, capacity of cylindrical tanker is more by $(14142.8 - 11785.7) = 2357.1$ L

7. Given, pencils are cylindrical in shape.

Length of one pencil $(h) = 25$ cm

and circumference of base, $2\pi r = 1.5$ cm

$$\Rightarrow \quad r = \frac{1.5 \times 7}{22 \times 2} = 0.2386 \text{ cm}$$

Now, curved surface area of one pencil $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 0.2386 \times 25$$

$$= \frac{262.46}{7} = 37.49 \text{ cm}^2$$

$$= \frac{37.49}{100} \text{ dm}^2 \qquad \left(\because 1 \text{ cm} = \frac{1}{10} \text{ dm}\right)$$

$$= 0.375 \text{ dm}^2$$

$\therefore$ Curved surface area of 120000 pencils

$$= 0.375 \times 120000$$

$$= 45000 \text{ dm}^2$$

Now, cost of colouring 1 dm^2 curved surface of the pencils manufactured in one day = ₹ 0.06

$\therefore$ Cost of colouring 45000 dm^2 curved surface

$$= 45000 \times 0.06 = ₹\ 2{,}700$$

8. Let r and h be the radius and height of a cone.

Given, radius of cone $(r) = 56$ cm

and curved surface area of cone = 12320 cm^2

$$\therefore \quad \pi r l = 12320$$

$$\Rightarrow \quad \frac{22}{7} \times 56 \times l = 12320$$

$$\Rightarrow \quad l = \frac{12320 \times 7}{22 \times 56} = \frac{86240}{22 \times 56} = 70$$

$$\Rightarrow \quad \sqrt{r^2 + h^2} = 70 \qquad (\because l^2 = r^2 + h^2)$$

$$\Rightarrow \quad (56)^2 + h^2 = (70)^2$$

$$\Rightarrow \quad h^2 = 4900 - 3136$$

$$= 1764$$

$$\Rightarrow \quad h = 42 \text{ cm}$$

Hence, height of a cone is 42 cm.

Short Answer Type-II Questions

1. Let r and h be the radius and height of the cone respectively.

Given, radius of cone = radius of hemisphere (r) = 7 cm

and height of the cone (h) = diameter of cone $= 2r = 2 \times 7 = 14$ cm

$\therefore$ Volume of the solid = Volume of cone + Volume of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

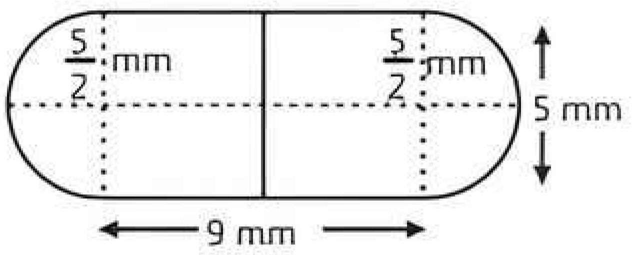
$$= \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times (14 + 2 \times 7)$$

$$= \frac{22 \times 7}{3} \times (14 + 14) = \frac{22 \times 7 \times 28}{3}$$

$$= \frac{4312}{3} = 1437.33 \text{ cm}^3$$

2.



From figure, radius of cylindrical part (r)

= Radius of hemispherical part (r)

$$= \frac{\text{Diameter of the capsule}}{2} = \frac{5}{2} \text{ mm}$$

Length of cylindrical part (h) = Length of the entire capsule $- 2 \times r = 14 - 5 = 9$ mm

∴ Surface area of capsule

= 2 × CSA of hemispherical part + CSA of cylindrical part

$$= 2 \times 2\pi r^2 + 2\pi rh$$

$$= 4\pi\left(\frac{5}{2}\right)^2 + 2\pi\left(\frac{5}{2}\right)(9) = 25\pi + 45\pi$$

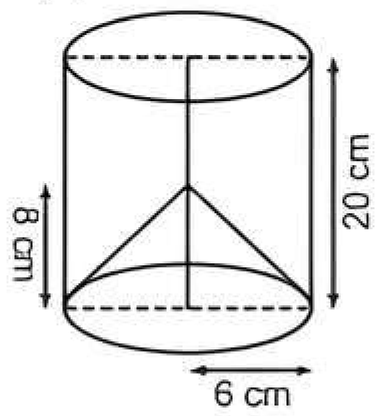
$$= 70\pi \text{ mm}^2 = 70 \times \frac{22}{7} \text{ mm}^2 = 220 \text{ mm}^2$$

Hence, the surface area of capsule is 220 mm^2.

3. Given, height of the solid cylinder (h) = 20 cm

height of the conical cavity (H) = 8 cm

and radius of conical base = radius of cylindrical base (r) = 6 cm



∴ Total surface area of the remaining solid = Curved surface area of conical cavity + Area of circular base of cylindrical part

$$= 2\pi rh + \pi rl + \pi r^2$$

$$= \pi r(2h + \sqrt{H^2 + r^2} + r)$$

$$= \frac{22}{7} \times 6 \times \left\{2 \times 20 + \sqrt{(8)^2 + (6)^2} + 6\right\}$$

$$= \frac{132}{7} \times \left\{40 + \sqrt{64 + 36} + 6\right\} = \frac{132}{7} \times \left\{40 + \sqrt{100} + 6\right\}$$

$$= \frac{132}{7} \times (40 + 10 + 6) = \frac{132 \times 56}{7} = 1056 \text{ cm}^2$$

COMMON ERR(!)R

In problems related to surface area and volume, students write incorrect formula and get wrong result.

4. Let 'r' be the radius of closed cylindrical oil tank.

Then, height of the tank $(h) = 6 \times r = 6r$ m ...(1)

Now, total surface area of closed cylinder

$$= 2\pi rh + 2\pi r^2$$

$$= 2\pi r(h + r) = \frac{2 \times 22r}{7}(6r + r)$$

$$= \frac{44}{7} \times 7r^2 = 44r^2 \text{ m}^2 \qquad \text{(From eq. (1))}$$

∵ Cost of painting the total outside surface of a closed cylindrical oil tank at 60 paise per m^2 = ₹ 237.60 (given)

$$\therefore \quad 44r^2 \times 0.60 = 237.60$$

$$\Rightarrow \quad r^2 = \frac{237.60}{44 \times 0.60} = \frac{23760}{44 \times 60}$$

$$\Rightarrow \quad r^2 = 9 \quad \Rightarrow \quad r = 3 \text{ m}$$

From eq. (1),

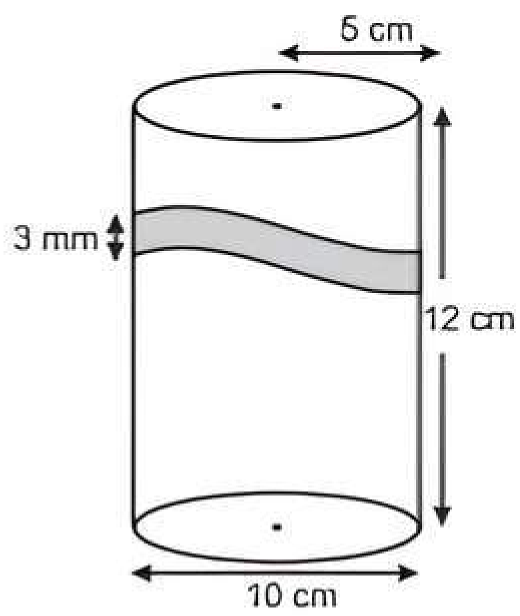
$$h = 6r = 6 \times 3 = 18 \text{ m}$$

Hence, the radius and height of the tank are 3 m and 18 m, respectively.

5. Given, diameter of the wire

$$= 3 \text{ mm} = \frac{3}{10} \text{ cm} \qquad \left(\because 1 \text{ mm} = \frac{1}{10} \text{ cm}\right)$$

∴ One round of the wire covers 3/10 cm of the height of the cylinder.



Also given,

Length of cylinder (h) = 12 cm

TiP

Adequate practice and remembering of formula is necessary.

Now, wire will cover entire length of cylinder.

$$\because \quad \frac{3}{10} \text{ cm is covered in} = 1 \text{ round}$$

$$\therefore \quad 1 \text{ cm is covered in} = 1 \times \frac{10}{3} \text{ rounds}$$

$$\therefore \quad 12 \text{ cm is covered in} = 12 \times \frac{10}{3} = 40 \text{ rounds}$$

Length of the wire required to complete one round

= Circumference of cylinder

$$= 2\pi r = 2\pi(5) = 10\pi \text{ cm}$$

So, length of the wire required to cover whole cylinder

= Length required to cover 1 round × Number of rounds

$$= 10\pi \times 40 = 400\pi = 400 \times 3.14 = 1256 \text{ cm}$$

Thus, required length of the wire is 1256 cm.

6. Given, diameter of the base of cone = 24 m

∴ Radius of the base of cone $(r) = 24/2 = 12$ m

Height of the cone $(h) = 3.5$ m

∵ A heap of rice is in the form of a cone.

∴ Volume of the rice = Volume of the conical heap

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (12)^2 \times 3.5 \text{m}^3$$

$$= \frac{22}{21} \times 12 \times 12 \times 3.5 \text{m}^3 = 528 \text{m}^3$$

Now, slant height of conical heap

$$(l) = \sqrt{h^2 + r^2} = \sqrt{(3.5)^2 + 12^2}$$

$$= \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5 \text{ m}$$

TR!CK

Quantity of canvas cloth is equal to the surface area of heap to just cover it.

∴ Curved surface area of the heap = $\pi r l$

$$= \frac{22}{7} \times 12 \times 12.5 \text{m}^2 = 471.43 \text{ m}^2$$

Hence, the required canvas cloth to just cover the heap is 471.43 m^2.

7. For Toy 1

Given, radius of hemisphere (r) = Radius of conical portion

= 3.5 cm

Height of conical portion $(h) = 15.5 - 3.5 = 12$ cm

Slant height of conical portion $(l) = \sqrt{h^2 + r^2}$

$$= \sqrt{(12)^2 + (3.5)^2} = \sqrt{144 + 12.25}$$

$$= \sqrt{156.25} = 12.5 \text{ cm}$$

∴ TSA of toy = CSA of cone + CSA of hemisphere

$$= \pi r l + 2\pi r^2 = \pi r (l + 2r)$$

$$= \frac{22}{7} \times 3.5 (12.5 + 2 \times 3.5)$$

$$= 11 \times 19.5 = 214.5 \text{ cm}^2$$

So, Cost of painting the toy = ₹ (1.50 × 214.5)

= ₹ 321.75

For Toy 2

Given, radius of hemisphere (r) = Radius of cylinder

$$= \frac{7}{2} \text{cm}$$

= 3.5 cm

Height of cylinder $(H) = 13 - 3.5 = 9.5$ cm

∴ Inner surface area of toy = CSA of hemisphere + CSA of cylinder

$$= 2\pi r^2 + 2\pi r H = 2\pi r(r + H)$$

$$= 2 \times \frac{22}{7} \times 3.5 (3.5 + 9.5)$$

$$= 22 \times 13 = 286 \text{ cm}^2$$

So, Cost of painting the toy = ₹ (1.50 × 286)

= ₹ 429

Hence, cost of painting the second toy is more by (429 – 321.75) = ₹ 107.25

8. Let the radius and height of cylinder be 'r' and 'h' respectively.

TiP

Students should learn formulae of basic figures and do adequate practice and also concentrate to the correct calculation.

Given, $r + h = 37$ cm ...(1)

and total surface area of cylinder = 1628 cm^2

$$\Rightarrow 2\pi r(r + h) = 1628$$

$$\Rightarrow 2\pi r(37) = 1628 \quad \text{(from eq. (1))}$$

$$\Rightarrow 2\pi r = \frac{1628}{37} = 44$$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 44$$

$$\Rightarrow r = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm}$$

∴ From eq. (1),

$$7 + h = 37$$

$$\Rightarrow h = 37 - 7 = 30 \text{ cm}$$

Hence, the volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 30$$

$$= 4620 \text{ cm}^3$$

9. Given, radius of cylinder (r) = Radius of cone $(r) = 5$ cm

Height of cylinder (h) = Height of cone $(h) = 12$ cm

∴ Slant height of the cone (l)

$$= \sqrt{r^2 + h^2}$$

$$= \sqrt{5^2 + 12^2} = \sqrt{25 + 144}$$

$$= \sqrt{169} = 13 \text{ cm}$$

So, Volume of the remaining solid

= Volume of the cylinder – Volume of the cone

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$= \frac{2}{3}\pi r^2 h = \frac{2}{3} \times 3.14 \times 5^2 \times 12 \text{ cm}^3 = 628 \text{ cm}^3$$

Now, total surface area of the remaining solid

= CSA of the cylinder + CSA of the cone + Area of the top of the cylinder

$$= 2\pi r h + \pi r l + \pi r^2 = \pi r(2h + l + r)$$

$$= 3.14 \times 5(2 \times 12 + 13 + 5) \text{ cm}^2$$

$$= 3.14 \times 5 \times 42 = 659.4 \text{ cm}^2$$

Hence, volume of the remaining solid is 628 cm^3 and its total surface area is 659.4 cm^2.

COMMON ERR(!)R

Sometimes students make mistake in finding the surface area of the remaining solid. They take surface area of solid = surface area of cylinder – surface area of cone, but it is wrong. The right way of finding surface area = Curve surface area of cylinder + surface area of cone + Area of top of cylinder.

10. Given, height of the glass, $(h) = 10$ cm

and inner diameter of the glass, $(d) = 5$ cm

∴ Inner radius of the glass, $(r) = \frac{d}{2} = \frac{5}{2} = 2.5$ cm

Now, apparent capacity of glass

$= \text{Volume of cylinder} = \pi r^2 h$

$= 3.14 \times (2.5)^2 \times 10$

$= 3.14 \times 6.25 \times 10 = 196.25 \text{ cm}^3$

Now, volume of hemispherical raised portion

$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times 3.14 \times (2.5)^3$

$= \frac{2}{3} \times 3.14 \times 15.625 = 32.71 \text{ cm}^3$

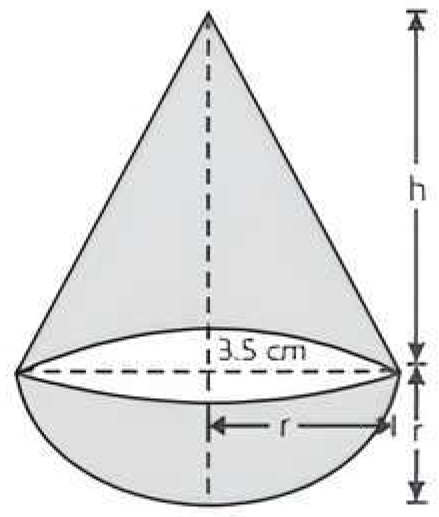
$\therefore$ The actual capacity of the glass

= Apparent capacity of glass – Volume of hemispherical raised portion

$= 196.25 - 32.71 = 163.54 \text{ cm}^3$

Long Answer Type Questions

1. Let the height of the conical part be 'h' cm.



Given, radius of cone (r) = radius of hemisphere

$(r) = 3.5$ cm

$\because$ Volume of the wood used to make the toy

$= 166\frac{5}{6} \text{ cm}^3$

$\therefore$ Volume of hemispherical part + Volume of the conical part

$= \frac{1001}{6} \text{ cm}^3$

TiP

Students should learn formulae of basic figures and do adequate practice and do correct calculations.

$\Rightarrow \quad \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = \frac{1001}{6}$

$\Rightarrow \quad \frac{1}{3}\pi r^2(2r + h) = \frac{1001}{6}$

$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 (2 \times 3.5 + h) = \frac{1001}{6}$

$\Rightarrow \quad 7 + h = \frac{1001}{6} \times \frac{3 \times 7}{22 \times (3.5)^2}$

$\Rightarrow \quad 7 + h = 13$

$\therefore \quad h = 13 - 7 = 6 \text{ cm}$

So, the height of toy $= h + r = 6 + 3.5 = 9.5$ cm

Now, Area to be painted = CSA of the hemisphere

$= 2\pi r^2$

$= 2 \times \frac{22}{7} \times (3.5)^2 \text{ cm}^2 = 77 \text{ cm}^2$

$\because$ Cost of painting 1 cm^2 area = ₹ 10

Hence, the cost of painting 77cm^2 area = ₹ (77×10)

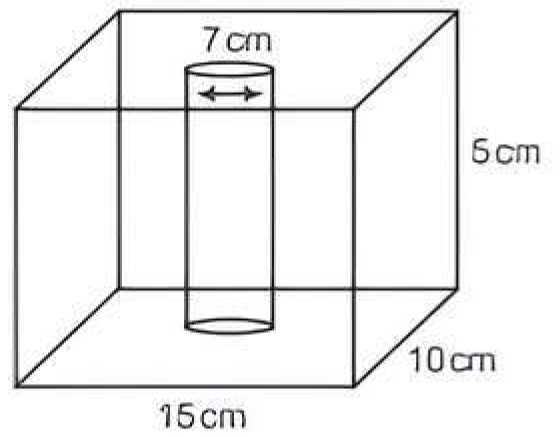
= ₹ 770

2. In cuboidal block,

length $l = 15$ cm

breadth $b = 10$ cm

and height $h = 5$ cm



$\therefore$ Surface area of the cuboidal block

$= 2(lb + bh + hl)$

$= 2(15 \times 10 + 10 \times 5 + 5 \times 15)$

$= 2(150 + 50 + 75) = 2 \times 275 = 550 \text{ cm}^2.$

Given diameter of cylindrical hole is 7 cm and its height $h = 5$ cm.

$\therefore$ Its radius $(r) = \frac{7}{2}$ cm

Curved surface area of cylindrical hole $= 2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 5 = 110 \text{ cm}^2$

Area of two circular base of cylindrical hole

$= \pi r^2 + \pi r^2 = 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = 77 \text{ cm}^2$

$\therefore$ The surface area of the remaining block

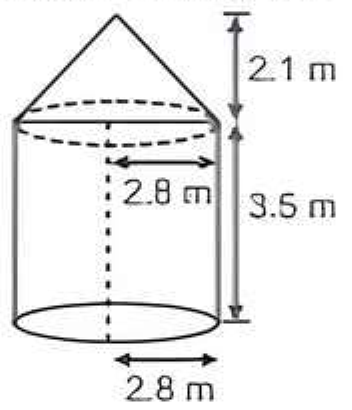
= Surface area of cuboidal block

+ CSA of cylindrical hole

– area of two circular bases of cylindrical hole

$= 550 \text{ cm}^2 + 110 \text{ cm}^2 - 77 \text{ cm}^2 = 583 \text{ cm}^2.$

3. Given, radius of the base of cylinder $(r) = 2.8$ m = Radius of the base of the cone (r).



Height of the cylinder $(h) = 3.5$ m

Height of the cone $(H) = 2.1$ m.

Slant height of conical part

$(l) = \sqrt{r^2 + H^2} = \sqrt{(2.8)^2 + (2.1)^2}$

$= \sqrt{7.84 + 4.41} = \sqrt{12.25} = 3.5$ m

Area of canvas used to make tent

= CSA of a cylinder + CSA of cone

$= 2\pi rh + \pi rl$

$= 2 \times \frac{22}{7} \times 2.8 \times 3.5 + \frac{22}{7} \times 2.8 \times 3.5$

$= 61.6 + 30.8 = 92.4 \text{ m}^2$

Cost of 1500 tents at ₹ 120 per sq. m

$= 1500 \times 120 \times 92.4 =$ ₹ 16,632,000

Share of each school to set up the tends

$= 16632000/50 =$ ₹ 332,640

4. Here, solid iron pole is a combination of two cylinders.

For first cylinder,

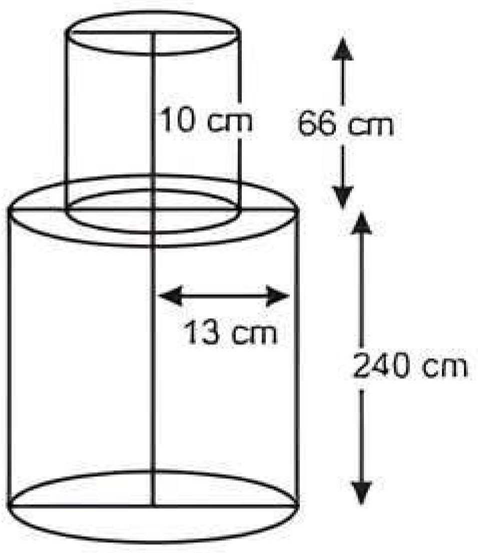
Height $(h_1) = 240$ cm

Base diameter = 26 cm

$\therefore$ Base radius $(r_1) = \dfrac{26}{2}$ cm = 13 cm

For second cylinder, height $(h_2) = 66$ cm
Radius $(r_2) = 10$ cm

$\because$ Volume of cylinder $= \pi r^2 h$



$\therefore$ Total volume of iron pole = Volume of first cylinder + Volume of second cylinder

$= \pi r_1^2 h_1 + \pi r_2^2 h_2$
$= \pi (13)^2 \times 240 + \pi (10)^2 \times 66$
$= \pi (169 \times 240 + 100 \times 66)$
$= 3.14 (40560 + 6600) = 3.14 \times 47160$
$= 148082.4 \text{ cm}^3$

Hence, total mass of the iron pole

$= 148082.4 \times 8 \text{ g} = 1184659.2 \text{ g}$ (given, $1 \text{ cm}^3 = 8$ g)

$= \dfrac{1184659.2}{1000}$ kg

$= 1184.66$ kg $\left(\because 1 \text{ g} = \dfrac{1}{1000} \text{ kg}\right)$

5. Given, rocket is the combination of a right circular cylinder and a cone.

Also, diameter of the cylinder = 6 cm

$\therefore$ Radius of the cylinder $(r) = \dfrac{6}{2} = 3$ cm

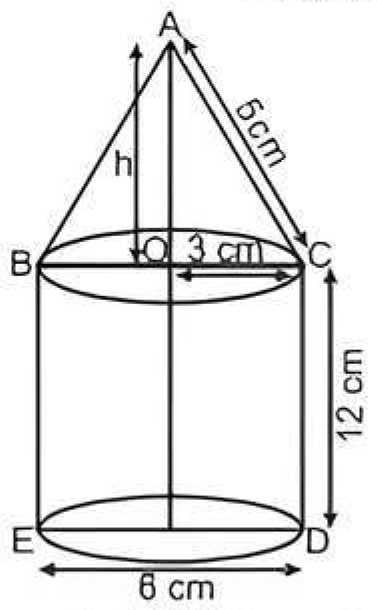
and height of the cylinder $(h) = 12$ cm

Now area of base of cylinder

$= \pi r^2 = 3.14 \times (3)^2 = 3.14 \times 9 = 28.26 \text{ cm}^2$.

$\therefore$ Volume of the cylinder $= \pi r^2 h = 3.14 \times (3)^2 \times 12$
$= 339.12 \text{ cm}^3$

and curved surface area $= 2\pi rh$
$= 2 \times 3.14 \times 3 \times 12$
$= 226.08 \text{ cm}^2$



Now, in right angled ΔAOC, use Pythagoras theorem

$$h = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$$

$\therefore$ Height of the cone, $H = 4$ cm

and radius of the cone, $r = 3$ cm

Now, volume of the cone $= \dfrac{1}{3}\pi r^2 H = \dfrac{1}{3} \times 3.14 \times (3)^2 \times 4$

$= \dfrac{113.04}{3} = 37.68 \text{ cm}^3$

and curved surface area of cone

$= \pi rl = 3.14 \times 3 \times 5 = 47.1 \text{ cm}^2$

Hence, total volume of the rocket

= volume of cylinder + volume of cone

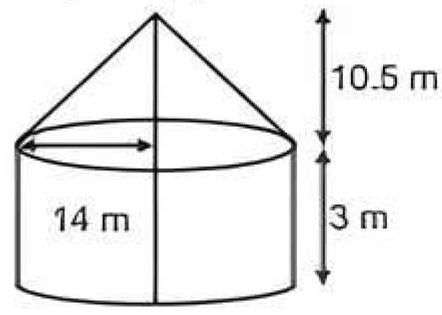
$= 339.12 + 37.68 = 376.8 \text{ cm}^3$

and total surface area of the rocket

= CSA of cone + CSA of cylinder + Area of base of cylinder

$= 47.1 + 226.08 + 28.26 = 301.44 \text{ cm}^2$

6. Given, height of the cylindrical part $(h) = 3$ m and radius of the cylindrical part $(r) = 14$ m



$\therefore$ Curved surface area of cylindrical part

$= 2\pi rh = 2 \times \dfrac{22}{7} \times 14 \times 3$

$= 264 \text{ m}^2$

Also, given total height of the tent $(H') = 13.5$ m

$\therefore$ Height of the conical part $(H) = H' - h$
$= 13.5 - 3 = 10.5$ m

and radius of conical part

= radius of cylindrical part $(r) = 14$ m

$\therefore$ Curved surface area of conical part

$= \pi rl = \pi r\sqrt{r^2 + H^2}$

$= \dfrac{22}{7} \times 14 \times \sqrt{(14)^2 + (10.5)^2}$

$= 44 \times \sqrt{196 + 110.25}$

$= 44 \times \sqrt{306.25} = 44 \times 17.5$

$= 770 \text{ m}^2$

$\therefore$ Total surface area of tent = CSA of cylindrical part + CSA of conical part $= 264 + 770 = 1034 \text{ m}^2$

$\because$ Provision for stitching and wastage $= 26 \text{ m}^2$

$\therefore$ Required area of canvas for making the tent

$= 1034 + 25 = 1060 \text{ m}^2$

So, cost of the canvas to be purchased

= Rate × TSA = ₹ 500 × 1060 = ₹ 5, 30,000

7. Given, length of the pond $(l) = 50$ m,

width of the pond $(b) = 44$ m

and water level is to rise by $(h) = 21$ cm

$= \dfrac{21}{100}$ m

$\therefore$ Volume of water in the pond $= l \times b \times h$

$= 50 \times 44 \times \dfrac{21}{100} = 462 \text{ m}^3$

Also given, diameter of the pipe = 14 cm

$\therefore$ Radius of the pipe $(r) = \dfrac{14}{2} = 7$ cm $= \dfrac{7}{100}$ m

Now, area of cross-section of pipe $= \pi r^2$

$$= \frac{22}{7} \times \left(\frac{7}{100}\right)^2 = \frac{154}{10000} m^2$$

and rate at which the water is flowing through the pipe, $h = 15$ km/h $= 15000$ m/h.

$\therefore$ Volume of water flowing in 1 hour = Area of cross-section × height of water coming out of pipe.

$$= \frac{154}{10000} \times 15000 = 231 m^3$$

$\therefore$ Time required to fill the pond

$$= \frac{\text{Volume of the pond}}{\text{Volume of water flowing in 1 hour}}$$

$$= \frac{462}{231} = 2 \text{ hours}$$

Let the rate at which the water is flowing through the pipe x km/h.

Now,

Time required to fill the pond × volume of water flowing in 1 hour = volume of the pond

1 × area of cross-section × height of water coming out of pipe = 462

$$\Rightarrow \frac{154}{10000} \times x = 462$$

$$\Rightarrow x = \frac{462 \times 10000}{154} = 30000 \text{ m/h}$$

$$= \frac{30000}{1000} \text{km/h} = 30 \text{ km/h}$$

Hence, speed of water if the rise in water level is to be attained in 1 hour = 30 km/h.

8. Given, the height of the cylinder $h = 10$ cm and the base radius of cylinder $r = 3.5$ cm

Also, the base radius of hemisphere

= base radius of cylinder (r)

= 3.5 cm.

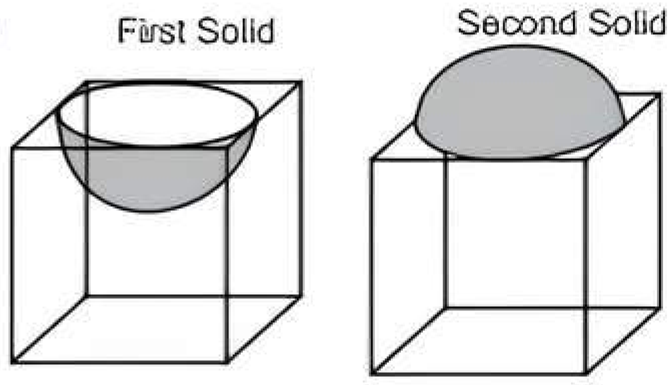
$\therefore$ Total surface area of the article = curved surface area of cylinder + 2 × curved surface area of hemisphere

$$= 2\pi rh + 2 \times 2\pi r^2 = 2\pi r(h + 2r)$$

$$= 2 \times \frac{22}{7} \times 3.5 \times (10 + 2 \times 3.5)$$

$$= \frac{44}{7} \times 3.5 \times 17 = 374 \text{ cm}^2$$

9. (i)



TiP

In both solid figures, the diameter of a hemisphere is equal to the side of a cubical box.

Given, edge of both identical cubical box = 7 cm

Diameter of both identical hemisphere = Edge of cubical box = 7 cm

$\Rightarrow$ Radius of both identical hemisphere,

$$(r) = \frac{7}{2} \text{ cm} = 3.5 \text{ cm}$$

Surface area for first new solid (S_1)

= Surface area of cube + surface area of hemisphere − area of circle

$$= 6 (\text{Side})^2 + 2\pi r^2 - \pi r^2$$

$$= 6 \times 7 \times 7 + 2 \times \frac{22}{7} \times (3.5)^2 - \frac{22}{7} \times (3.5)^2$$

$$= 294 + 77 - 38.5 = 332.5 \text{ cm}^2$$

Surface area for second new solid (S_2)

$$= 6 (\text{Side})^2 + 2\pi r^2 - \pi r^2$$

$$= 6 \times 7 \times 7 + 2 \times \frac{22}{7} \times (3.5)^2 - \frac{22}{7} \times (3.5)$$

$$= 294 + 77 - 38.5 = 332.5 \text{ cm}^2$$

$\therefore S_1 : S_2 = 1 : 1$

(ii) Volume for first new solid (V_1) = Volume of cube − Volume of hemisphere

$$= (\text{Side})^3 - \frac{2}{3}\pi r^3$$

$$= 7 \times 7 \times 7 - \frac{2}{3} \times \frac{22}{7} \times (3.5)^3 = 343 - \frac{539}{6} = \frac{1519}{6} \text{ cm}^3$$

$$= 253.17 \text{ cm}^3$$

Volume for second new solid

(V_2) = Volume of cube + Volume of hemisphere

$$= (\text{Side})^3 + \frac{2}{3}\pi r^3 = 7 \times 7 \times 7 + \frac{2}{3} \times \frac{22}{7} \times (3.5)^3$$

$$= 343 + \frac{539}{6} = \frac{2597}{6} \text{ cm}^3$$

$$= 432.83 \text{ cm}^3$$

Chapter Test

Multiple Choice Questions

Q 1. Two different cylinders of same radius 7 cm but different heights 10 cm and 12 cm are joined along the base. The capacity of combined cylinder is: $\left[\text{Use } \pi = \frac{22}{7}\right]$

a. 3380 cm^3 b. 3388 cm^3

c. 3340 cm^3 d. 3345 cm^3

Q 2. Two cubes each of volume 8 cm^3 are joined end to end. Then the surface area of resulting cuboid is:

a. 48 cm^2 b. 44 cm^2

c. 40 cm^2 d. 50 cm^2

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): A toy is in the form of a cone of radius 3.5 cm surmounted on a hemisphere of same radius. If the total height of the toy is 15.5 cm, then volume of the toy is 243.83 cm^3. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Reason (R): The volume of combined figure can be determined by sum of the volumes of individual solid figure.

Q 4. Assertion (A): Two identical cubes of each of volume 64 cm^3 are joined together end to end. The surface area of the resulting cuboid is 350 cm^2.

Reason (R): The shape of a gilli, in the gilli-danda game is a combination of two cones and one cylinder.

Fill in the Blanks

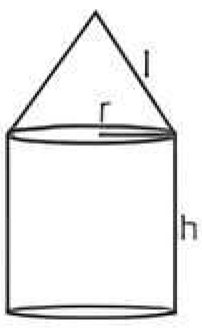
Q 5. A solid ball is exactly fitted inside the cubical box of side a. The volume of the ball is ....

Q 6. If a cone of height 12 cm is mounted on a hemisphere of same radius 5 cm, then the height of the combined figure is ....

True/False

Q 7. Solid cone is exactly fitted inside a cubical box of side a. The volume of the cone is $\frac{\pi}{12}a^3$.

Q 8. The curved surface area of the given solid figure is $2\pi rh + \pi rl$.



Case Study Based Question

Q 9. Teacher decided to take some students on a picnic trip to Mussoorie, where they saw Kempty falls, Lal Tibba, company garden etc. At night they hired a tent. The tent was in a shape of cylinder at the bottom and conical at the top and is made up of good quality fabric.

The diameter of the base of the cylindrical and conical portion is equal, which is 7 feet. The total height of the tent is equal to its diameter and the height of the cylindrical portion is 4 feet.

Based on the given information, solve the following questions:

(i) Find the surface area of the conical portion.

(ii) Find the total fabric is used to make a tent.

Or

Find the total capacity of the tent.

(iii) If each child occupies a surface area of $\frac{7}{2}$ sq. feet, then find the maximum number of children that can be accommodated in the tent.

Very Short Answer Type Questions

Q 10. Two cubes have their volumes in the ratio 1 : 27. What is the ratio of their surface areas?

Q 11. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and the radius of each hemispherical end is 7 cm, then find the cost of polishing its surface at the rate of ₹ 2 per dm^2. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Short Answer Type-I Questions

Q 12. Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed.

Q 13. 50 circular plates, each of radius 7 cm and thickness 0.5 cm are placed one above another to form a solid right circular cylinder. Find the volume of the cylinder so formed.

Short Answer Type-II Questions

Q 14. A sphere of maximum volume is cut-out from a solid hemisphere of radius r. What is the ratio of the volume of the hemisphere to that of the cut-out sphere?

Q 15. A building is in the form of a cylinder surmounted by a hemispherical dome. The base diameter of the dome is equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building, if it contains $67\frac{1}{21}$ m^3 of air.

Long Answer Type Question

Q 16. A solid iron-pole having cylindrical portion 110 cm high and of base diameter 12 cm is surmounted by a cone 9 cm high. Find the mass of the pole, given that the mass of 1 cm^3 of iron is 82 gm.